Supplementary Material 1

It is tempting to use a statistic to represent the variations in the relative signal intensity shown in Figure 1. The amplitude scintillation index S_4 is the ratio of the standard deviation of the signal intensity in a given time interval σ_I to the mean value of the signal intensity in that interval $\langle I \rangle$ (Briggs and Parkin, 1963). It is calculated as:

$$S_4 = \frac{\sigma_I}{\langle I \rangle} \quad \text{(S1)}$$

This statistic is commonly used for studies of scintillation of signals used by GNSS at frequencies in the GHz range, with S_4 often calculated over a time interval of one minute (e.g. de Franceschi et al., 2019), however, it is also used at other frequencies. Briggs and Parkin (1963) originally used this index in a paper covering a number of radio sources including observations of Cassiopeia A at 38 MHz. In order for S_4 to be an appropriate metric for the variations observed, the time interval over which it is calculated must exceed the duration of the variations to be considered. Some of the ripples observed in Figure 1, particularly at low frequencies immediately before the depletion, have durations approaching one minute. Therefore, S_4 has been calculated from the data shown in Figure 1 for a range of time intervals between 10 seconds and 10 minutes in ten second increments. The results for three frequencies, 30.7 MHz, 44.3 MHz and 58.0 MHz, which correspond to the 30th, 100th and 170th frequency channel in the dynamic spectrum, are shown in Figure S1. These channels were chosen to span the range of frequencies within the observation. A summary plot showing how the maximum value of S_4 at different time intervals at frequencies between 30.7 MHz and 61.9 MHz in increments of 1.95 MHz (10 channels) is also shown, with the lowest frequencies present in the dynamic spectra (Figure 1) excluded as channels were removed due to being contaminated by noise. It is readily apparent that the value of the S_4 index depends upon both the time interval and frequency. Simply choosing the time corresponding to the maximum value of S_4 may seem to be an appealing solution, for example, at a frequency of 44.5 MHz the largest value of S_4 is 0.32 and is recorded at a time interval of 30 seconds, which corresponds to the longest period ripple which was observed. However, care should be taken with such an approach. At the larger time intervals, the calculation of S_4 uses data over a sufficiently long time interval that it is not representative of the observed variations. For example, it is clear from the dynamic spectra that, at a time of 17:50 UT and a frequency of 58.0 MHz, the variations in the received signal intensity are relatively low. However, if S_4 is calculated over a time interval of over 6.5 minutes, elevated values are observed due to the ripples which border the depletion in Figure 1. Furthermore, while the maximum value of S_4 (which took a value of 0.42) is also calculated over a time interval of 30 seconds at this frequency, there is also a local maximum of 0.32 at a time interval of 9 minutes 10 seconds. The maximum value of S_4 increases with frequency, whereas the opposite is normally the case. This is a likely consequence of the varying sensitivity of the LBA, which peaks at around 58 MHz (van Haarlem et al., 2013).

The variations in the relative intensity of the received signal are caused by irregularities with a spatial scale size ranging from the Fresnel dimension to an order of magnitude below this value (Basu et al., 1998). The Fresnel length D_F is related to the wavelength of the received signal λ and the line of sight distance from the receiver to the scattering region L:

$$D_F = \sqrt{2 \cdot \lambda \cdot L} \quad \text{(S2)}$$

It is apparent that the variations in the intensity of the received signal are related to both the scale size of the plasma structure causing the variations and the line of sight distance to this structure. Rather than using the maximum values of S_4, a more useful approach may be to begin by determining both the Fresnel scale and velocity. It would then be feasible to calculate S_4 across an

appropriate timescale. Such an approach was used by Flisek et al. (2023) on a case study basis, when S_4 was calculated over a time interval of three minutes. If the observations were then to be detrended on this timescale, S_4 could then be calculated on the detrended signal and use as a proxy for the noise level. As a different time interval may be more appropriate under other conditions, further work is needed before S_4 can be routinely used as a proxy for the variations in the relative signal intensities due to refractive effects at these frequencies.

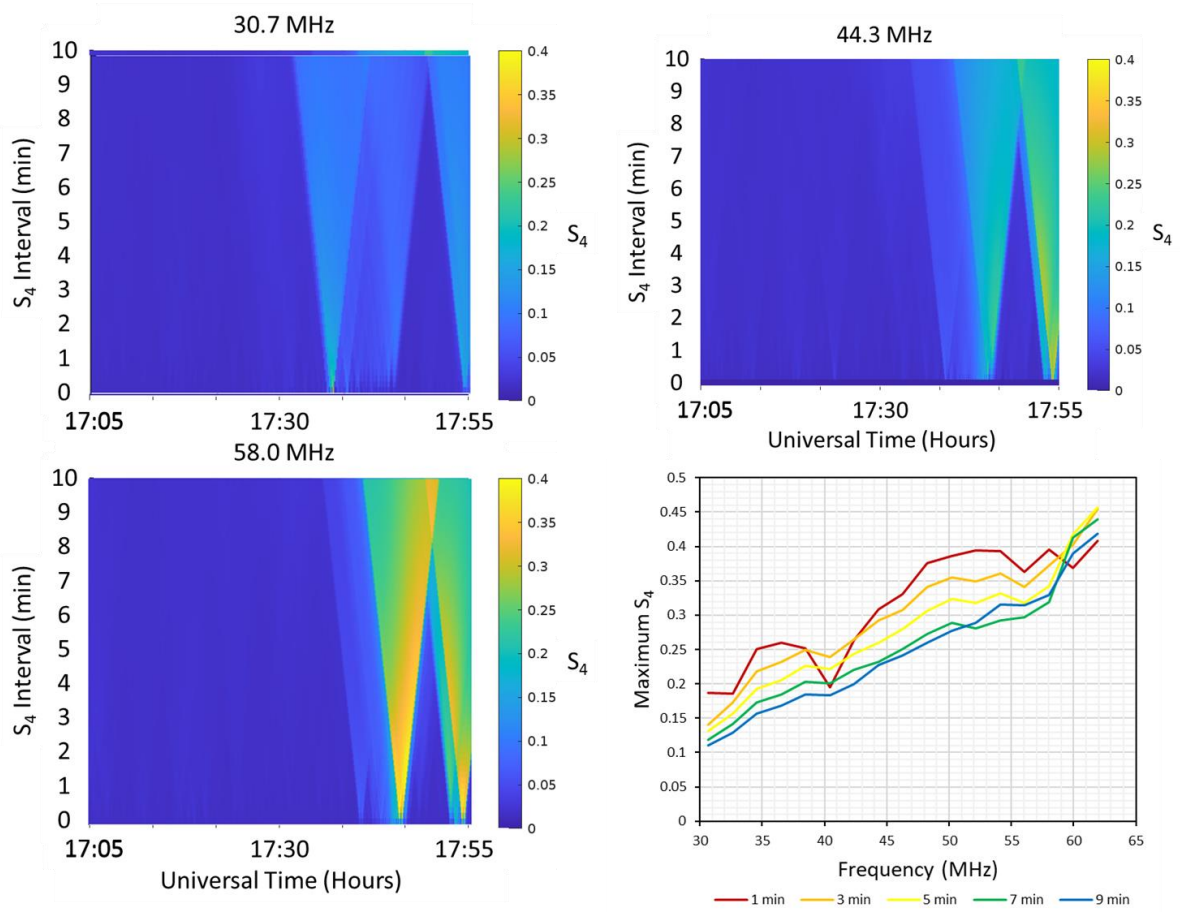

Figure S1: The amplitude scintillation index S_4 as a function of time and the interval over which it was calculated for an observation of Cygnus A made by LOFAR station RS508 on 14th July 2018 between 17:05 UT and 17:55 UT. Three frequencies are shown, 30.7 MHz, 44.3 MHz and 58.0 MHz, with these values chosen as they correspond to the 30th, 100th and 170th frequency channel in the dynamic spectra. Also shown is a summary plot showing the maximum value of S_4 at different time intervals at frequencies between 30.7 MHz and 61.9 MHz in increments of 1.95 MHz (10 channels)