

On the parametrization of the energetic-particle pitch-angle diffusion coefficient

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ABSTRACT

Context: Solar energetic particle (SEP) events are one of the key ingredients of the near-Earth radiation environment. Pitch-angle scattering by fluctuations imposed on the large-scale magnetic field is assumed to be the basic physical process behind diffusive propagation of SEPs in the heliosphere. Various pitch-angle diffusion models have been suggested to parametrize the wave-particle interactions, based on the original results of the classical quasi-linear theory of particle scattering and improved new approaches.

Aims: We investigate under which circumstances the different functional forms of the pitch-angle diffusion coefficient can lead to equivalent results. In particular, we use two forms that are commonly used in two types of numerical methods to solve the particle transport equation, i.e., finite difference methods and Monte Carlo simulations.

Methods: We estimate the corresponding values of the parameters of the two scattering models by performing a least-square fitting of the functional form of one of the scattering-frequency models to the other. We also perform Monte Carlo simulations of near-relativistic solar electrons to investigate the similarity of the models in terms of observables at 1 AU.

Results: Our study shows that the two forms of pitch-angle scattering frequency lead to nearly equivalent results for electron transport from the Sun to 1 AU. We give the equivalent scattering parameters of the two models as curves that can be easily used when comparing the results of the two models.

Conclusions: By providing the equivalent parametrizations of two commonly used scattering models, we provide key information on how to relate the results from the two parametrizations to each other and to the theory of particle transport.

Key words. methods: numerical – interplanetary medium – Sun: particle emission

1. Introduction

Interactions between charged particles and turbulent magnetic fluctuations carried by space plasmas constitute a fundamental process governing the propagation of energetic particles in the space environment (see, e.g., Schlickeiser 2002, for a comprehensive treatment). As charged-particle radiation is one of the main components of space weather (Watermann et al. 2009), forecasting the temporal development of the near-Earth radiation environment is one of the main goals of space weather modeling (Vainio et al. 2009). Thus, one of the most important building blocks of radiation environment modeling is a charged particle transport tool that allows computations of the evolution of radiation intensities from known sources at the Sun, in the heliosphere and in the planetary environments.

Modeling of solar energetic particle (SEP) events provides us with the possibility to test the validity of different theories of the interaction of charged particles with magnetic fields. In the absence of large-scale disturbances such as coronal mass ejections (CMEs) and shocks, the interplanetary magnetic field (IMF) can be described as a smooth average field, represented by an Archimedean spiral, with superposed magnetic fluctuations. In this case, the propagation of SEPs along the IMF has two components, adiabatic motion along the smooth field and pitch-angle scattering by magnetic turbulence. This transport model is referred to as focused transport.

The classical quasi-linear theory of particle scattering (standard QLT; Jokipii 1966) predicts a pitch-angle diffusion coefficient related not only to the level but also to the power spectrum of the magnetic field fluctuations. In standard QLT, pitch-angle scattering is governed by a pitch-angle diffusion coefficient of the form $D_{\mu\mu} = v_0 (1 - \mu^2)|\mu|^{q-1}$, where $\mu = \cos \theta$ is the particle pitch-angle cosine, q is the spectral slope of the magnetic field power spectrum, and v_0 is determined by the overall level of fluctuations. Note that values of $q > 2$ at the highest wavenumbers yield the standard-QLT prediction that particles cannot scatter across $\mu = 0$ at all, which is not supported by observations.

Over the years several additions and refinements to QLT have been proposed to give a better explanation of the observations (e.g., Vainio 2000). A pragmatic approach to this problem is simply introducing finite values of $D_{\mu\mu}$ for $\mu = 0$. Beek & Wibberenz (1986) proposed a modified form of the pitch-angle diffusion coefficient given by $v(\mu) = v_0(|\mu|^{q-1} + H)$, which partially resembles the result of the standard theory (corresponding to $H = 0$) and additionally introduces a parameter H to phenomenologically describe an enhancement of scattering through $\mu = 0$ by non-resonant and nonlinear effects. This form of the pitch-angle diffusion coefficient is used in many numerical particle transport codes employing the finite difference method to solve the equation of focused transport (e.g., Dröge & Kartavykh 2009).

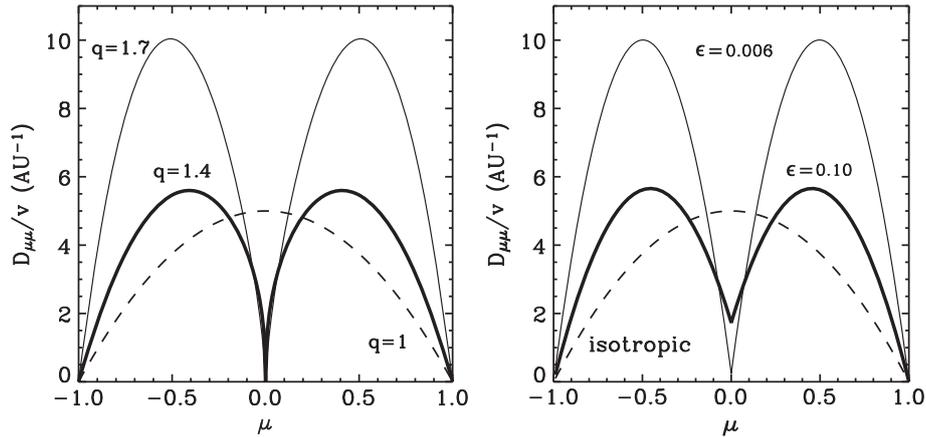


Fig. 1. Pitch-angle diffusion coefficients for $\lambda_{\parallel} = 0.1$ AU; for $H = 0$ and different slopes q of the power spectrum of the fluctuating magnetic field (*left*) and different values of the ϵ parameter (*right*).

The application of the Monte Carlo method in the propagation of charged particles in the heliosphere dates back to the 1970s (e.g., Jokipii & Levy 1977; Jokipii & Kopriva 1979). The early applications treated the energetic charged particles as random walkers in the configuration space in the spirit of the Parker equation (Parker 1965). While the method continues to be used to solve the cosmic-ray transport equation in the heliosphere (e.g., Gervasi et al. 1999; Bobik et al. 2012), the method was applied to focused transport already in the early 1980s (Palmer & Jokipii 1981; Earl & Jokipii 1985). Modern Monte Carlo models of focused transport are now routinely applied to interplanetary transport and acceleration of SEPs (e.g., Kocharov et al. 1998; Vainio 1998; Agueda et al. 2009; Dröge et al. 2010; Wang et al. 2012).

The advantage of the Monte Carlo method is that it is very flexible in terms of physical processes that can be included. Processes like advection and adiabatic deceleration due to solar wind expansion (Ruffolo 1995; Kocharov et al. 1998) and perpendicular diffusion (Dröge et al. 2010) are most easily treated in Monte Carlo simulations, as typically their inclusion adds but some tens of lines of code to the solver. The Monte Carlo model has even been applied to full-orbit calculations resolving the gyromotion of SEPs in the large-scale magnetic field (e.g., Pei et al. 2006; Sandroos & Vainio 2009).

The scattering operators in Monte Carlo solvers differ a lot in their efficiency. In principle, general Itô calculus with low-order methods, like the explicit Euler method, can be used, but their efficiency is low since the time steps have to be kept quite small to achieve accurate results on the form of the pitch-angle distribution (e.g., Vainio 1998). Specialized methods, based on exact or almost exact solutions of the pitch-angle diffusion equation, can be orders of magnitude more efficient than the general numerical methods. A well-known method (e.g., Torsti et al. 1996; Kocharov et al. 1998) exists for isotropic scattering (i.e., $v = \text{const.}$), which is based on an analytical solution of the diffusion equation on a spherical surface. However, as isotropic scattering is but a special case of pitch-angle diffusion, a need for efficient but more general methods in dealing with pitch-angle diffusion is evident.

For practical purposes, Agueda et al. (2008) proposed a scattering frequency of the form $\nu(\mu) = \nu_0 \left(\frac{|\mu|}{1+|\mu|} + \epsilon \right)$, where ϵ is the only parameter that regulates the shape of the pitch-angle diffusion coefficient. Several simulation studies have assumed this functional form (Agueda et al. 2008, 2009, 2010). This is numerically very advantageous, as it reduces to the application of the

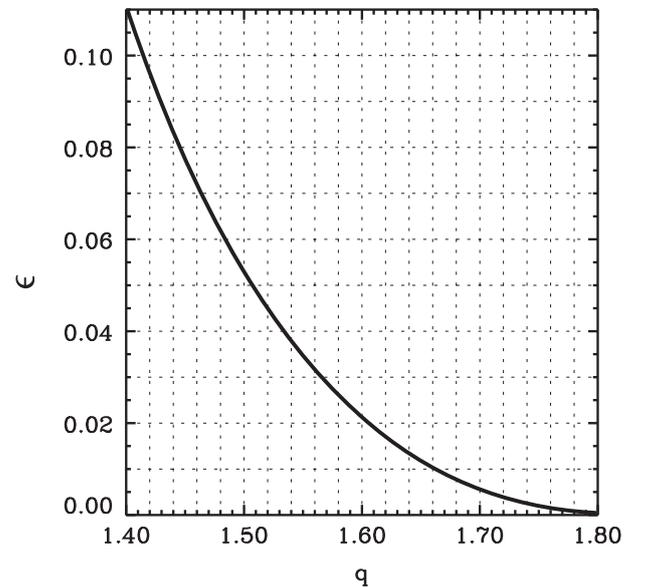


Fig. 2. Corresponding values of ϵ and q for the standard pitch-angle diffusion coefficient ($H = 0$).

isotropic scattering method after a coordinate transformation. However, so far the method has not been investigated in terms of its potential to model scattering off magnetic fluctuations with different types of power spectra. In particular, it is of interest to investigate, whether the anisotropic scattering method, with a proper choice of parameters, could actually approximate the models based on QLT and its extensions. In the present work we investigate under which circumstances the different functional forms of the pitch-angle diffusion coefficients can lead to equivalent results. In Section 2 we review the details of the scattering models. In Section 3 we present the corresponding values of q , H , and ϵ . We summarize this work in Section 4.

2. Parametrization of the pitch-angle diffusion coefficient

The focused transport equation governs the evolution of the particle's phase space density $f(s, \mu, t)$ (Roelof 1969)

$$\begin{aligned} \frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} + \frac{1 - \mu^2}{2L} v \frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu} \left(D_{\mu\mu}(\mu) \frac{\partial f}{\partial \mu} \right) \\ = q(z, \mu, t) \end{aligned} \quad (1)$$

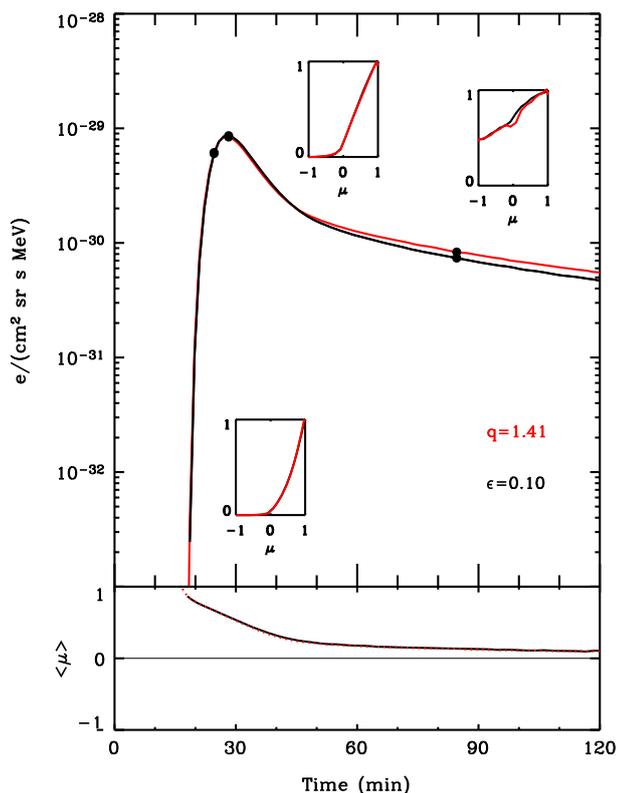
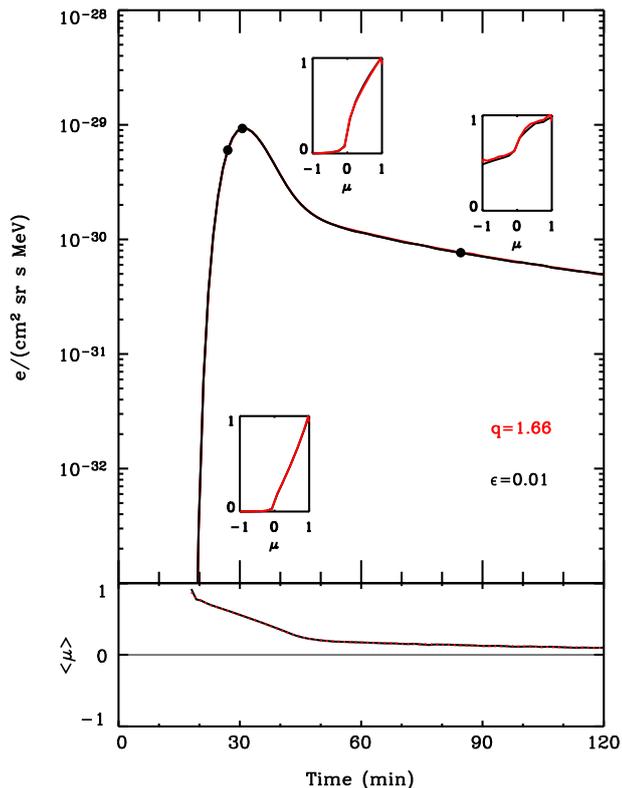


Fig. 3. Green's functions of interplanetary transport for $\lambda_r = 0.5$ AU, for two scattering cases: case A ($\epsilon = 0.01$, $q = 1.66$) and case B ($\epsilon = 0.10$, $q = 1.41$). From top to bottom: 100–180 keV omnidirectional intensity and mean pitch-angle cosine. Normalized pitch-angle distributions are displayed for three snapshots (black dots).

where z is the distance along the magnetic field line, μ is the particle pitch-angle cosine, and t is the time. The IMF systematic effect is characterized by the focusing length

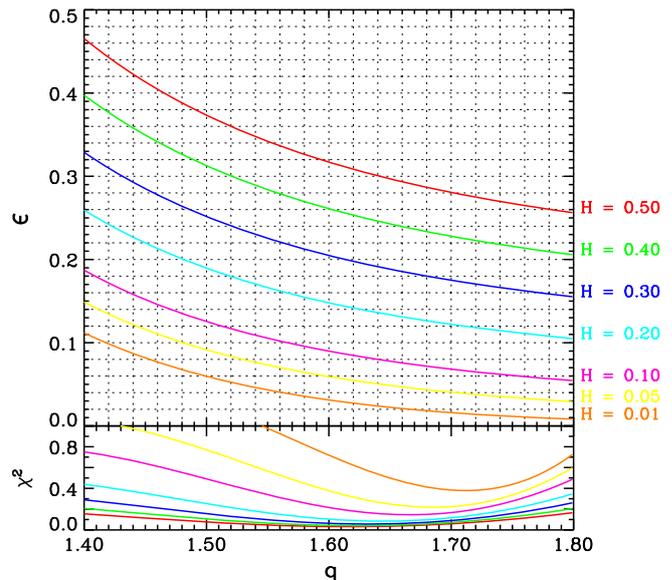


Fig. 4. From top to bottom: for seven values of H , corresponding q and ϵ values estimated by fitting the form of the modified standard pitch-angle diffusion coefficient with the ϵ -model. Sum of the squared differences between the two pitch-angle diffusion coefficients.

$L(z) = B(z)/(-\partial B/\partial z)$ in the diverging magnetic field B , and the stochastic forces are described by the pitch-angle diffusion coefficient $D_{\mu\mu}$. The injection of particles close to the Sun is given by $q(z, \mu, t)$. Equation (1) neglects convection and adiabatic deceleration (see Ruffolo 1995, for the full equation).

As analytical solutions of the transport equation are not known, numerical methods have to be applied to solve Eq. (1) (e.g., Ruffolo 1995; Kocharov et al. 1998; Lario et al. 1998; Dröge 2003; Agueda et al. 2008).

Assuming QLT and fluctuations that are transverse and axially symmetric (slab model), Jokipii (1966) explicitly derived the pitch-angle diffusion coefficient in terms of the power spectrum of the fluctuating field. The combination of the QLT with the slab model is known as the standard model of particle scattering (Jokipii 1966; Jaekel & Schlickeiser 1992).

If the power spectrum of the fluctuating field can be represented by a power law, $P(k) \propto k^{-q}$, where k is the wavenumber parallel to the magnetic field and q is the spectral slope of the magnetic field power spectrum, then the standard model predicts a pitch angle diffusion coefficient of the form:

$$D_{\mu\mu} = \frac{v(\mu)}{2} (1 - \mu^2) \quad (2)$$

where v is the scattering frequency given by $v(\mu) = v_0 |\mu|^{q-1}$, with $v_0 = 6v/[2\lambda_{\parallel}(4-q)(2-q)]$. Under strong scattering conditions, the relation of $D_{\mu\mu}$ to the parallel scattering mean free path, λ_{\parallel} , is given by (Hasselmann & Wibberenz 1970)

$$\lambda_{\parallel} = \frac{3v}{8} \int_{-1}^1 \frac{(1 - \mu^2)^2}{D_{\mu\mu}} d\mu = \frac{3v}{4} \int_{-1}^1 \frac{1 - \mu^2}{v(\mu)} d\mu. \quad (3)$$

The intensity of scattering between the Sun and the observer is often characterized by a spatially constant radial mean free path (Kallenrode et al. 1992), λ_r , given by $\lambda_r = \lambda_{\parallel} \cos^2 \psi$, where ψ is the angle between the field line and the radial direction.

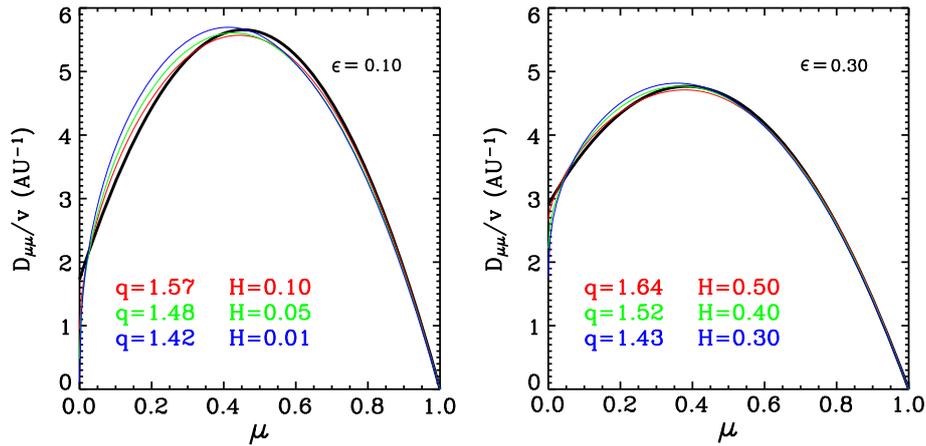


Fig. 5. Modified QLT diffusion coefficient for different combinations of q and H (thin colored curves) together with the corresponding ϵ -model diffusion coefficient (black curve), for $\lambda_r = 0.10$ AU.

Measured magnetic field spectra give spectral slopes in the range $1.3 \leq q \leq 1.9$ with an average value of $q = 1.63$ (Kunow et al. 1991). Figure 1 (left) shows the shape of the pitch angle diffusion coefficient for different slopes of the magnetic field power spectrum and $\lambda_{\parallel} = 0.1$ AU. The scattering is isotropic when $q = 1$, i.e., when the strength of the scattering is independent of the pitch-angle of the particles, $v = v_0$. As the power spectrum becomes steeper, particles with pitch-angles close to 90° experience less scattering; a gap develops around $\mu = 0$ with increasing slope of the magnetic field spectrum and particle transport becomes decoupled in two hemispheres (parallel and anti-parallel to the magnetic field vector) once the spectral index becomes $q = 2$.

Previous works (see, e.g., Beeck & Wibberenz 1986) have used a modified standard pitch-angle diffusion coefficient given by

$$D_{\mu\mu} = \frac{v_0}{2} (|\mu|^{q-1} + H)(1 - \mu^2), \quad (4)$$

where H is a parameter that describes the amount of scattering through $\mu = 0$. In this case, the form of the pitch-angle diffusion coefficient features reduced but finite scattering through $\mu = 0$, as predicted by current models of particle scattering (Dröge 2000). The introduction of this additional constant, H , serves to phenomenologically describe nonlinear corrections to the QLT case; that is, an enhancement of scattering through $\mu = 0$ by non-resonant and nonlinear effects.

Moreover, we observe solar events where the scattering near $\mu = 0$ is so small that it has to be described by large values of $q > 2$. In the case of QLT (Jokipii 1966) – corresponding to $H = 0$ – we would have totally coherent transport with no pitch-angle scattering across $\mu = 0$. This is not observed experimentally and this kind of behavior can be modeled by taking $q > 2$ combined with $H > 0$ (Beeck & Wibberenz 1986).

For practical purposes, Agueda et al. (2008) proposed a pitch-angle diffusion coefficient that partially resembles the result of the standard theory and features reduced but finite scattering through $\mu = 0$, by assuming the form

$$D_{\mu\mu} = \frac{v_0}{2} \left(\frac{|\mu|}{1 + |\mu|} + \epsilon \right) (1 - \mu^2), \quad (5)$$

where ϵ is the only parameter that allows us to consider a range of scattering conditions. The main motivation for this functional form of the pitch-angle diffusion coefficient is its very efficient implementation to Monte Carlo simulations.

3. Results

The different pitch-angle diffusion coefficients can be shown to be almost equivalent for a set of corresponding values of the model parameters.

3.1. Corresponding values of q and ϵ for $H = 0$

We estimated the corresponding values of the parameters q and ϵ by minimizing the sum of the squared differences between the standard pitch-angle diffusion coefficient ($H = 0$) and the diffusion coefficient for the ϵ -model. Figure 2 shows the corresponding values of the parameters q and ϵ . It can be seen that for a value of $q = 1.66$ it corresponds $\epsilon = 0.01$ (case A), while for $q = 1.41$, it corresponds a value of $\epsilon = 0.10$ (case B).

We use an interplanetary transport model to simulate the propagation of solar electrons along the Archimedean interplanetary magnetic field (Agueda et al. 2008). The model uses the Monte Carlo technique to solve the focused transport equation, including the effects of convection with scattering fluctuations and adiabatic deceleration resulting from the interplay of scattering and focusing (Ruffolo 1995). For the ϵ -parametrization it uses the fast specially devised method of Agueda et al. (2008) and for the q -parametrization a general stochastic differential equation solver. The model computes the directional distribution of electrons at the spacecraft location resulting from an instantaneous injection at $t = 0$ close to the Sun, i.e., it provides the Green's function of interplanetary transport.

Figure 3 displays the Green's functions of interplanetary transport assuming $\lambda_r = 0.5$ AU and the pitch-angle diffusion coefficients defined in case A and case B. It can be seen that the Green's functions are indistinguishable for case A, and there are some differences for case B during the decay phase of the event, that are smaller than 20%.

3.2. Corresponding values of q , H and ϵ

For a set of combinations of q and H , we calculated the corresponding values of ϵ by estimating the value that best fits the modified standard pitch-angle diffusion coefficient.

Figure 4 (top panel) shows the corresponding values of q and ϵ for several values of H , from 0.01 to 0.50. Bottom panel in Figure 4 shows the sum of the squared differences between

the modified standard pitch-angle diffusion coefficient and the pitch-angle diffusion coefficient of the ϵ -model. It can be seen that the larger the value of H , the more similar are the curves for a given value of q .

For $q = 1.67$ and $H = 0.05$ (set of parameters used by e.g., Dröge & Kartavykh 2008), the corresponding value of ϵ is 0.045. Figure 5 shows the form of the pitch-angle diffusion coefficients for different combinations of the parameters q and H , and the $D_{\mu\mu}$ for the corresponding value of ϵ .

Note that certain combinations of q and H lead to same value of ϵ , as a slight increase of q can be compensated by an increase in H in Eq. (4). As an example we show in Figure 5 the shapes of $D_{\mu\mu}(\mu)$ for two values of ϵ and their corresponding q and H combinations. As inferred from Figure 4, the two models are most in agreement for the largest value of H in each case.

4. Conclusions

Understanding the mechanisms by which energetic charged particles are scattered in turbulent magnetic fields continues to be one of the outstanding problems of modern heliophysics (Dröge 2000). In this paper we presented several pitch-angle diffusion coefficient models and their relation to the nature of magnetic field fluctuations and to the amount of nonlinear corrections to the QLT.

Early treatments of the scattering diffusion coefficient employed approximations which resembled the results of the QLT in a slab model, but with finite scattering through $\mu = 0$. Subsequent investigations have explored other parametrizations of the pitch-angle diffusion coefficient, including also the reduced but finite scattering through $\mu = 0$.

We showed that despite their different functional forms, two commonly used parametrizations of the pitch-angle diffusion coefficient are nearly equivalent and can be, for practical purposes, used interchangeably. We also gave the best-fit correspondence between the parametrization, which can be utilized when comparing results from the two scattering models. These results show that the development of transport simulations can proceed using the two different parametrizations without losing the possibility to compare their results with each other and with theory.

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